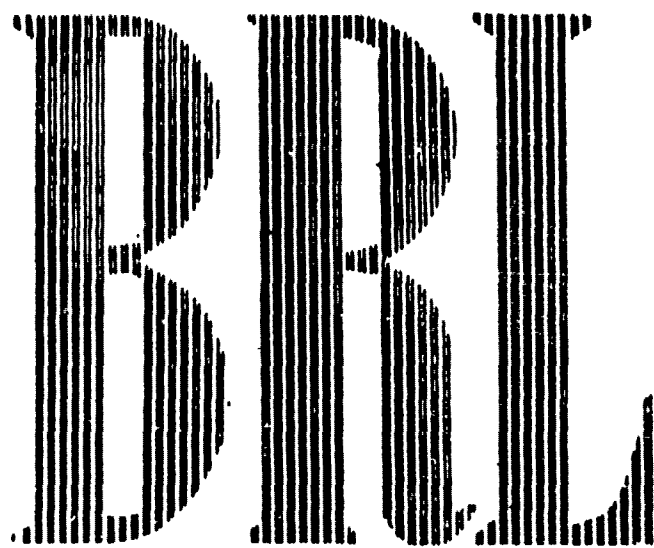


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MEMORANDUM REPORT NO. 1071
APRIL 1957

COMMENTS ON PROJECTILE JUMP

Charles H. Murphy

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CHMurphy/blf
Aberdeen Proving Ground, Md.
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ABSTRACT

Relations for aerodynamic jump and blast jump are derived from the linear theory. Although most of the conclusions are not new, the derivations provide additional insight into the problem.

TABLE OF SYMBOLS

A	axial moment of inertia
B	transverse moment of inertia
d	diameter
F_1, F_2, F_3	components of the aerodynamic force in the missile coordinate system
F_x, F_y, F_z	components of the aerodynamic force in the range coordinate system
$H = \frac{\rho d^3}{m} [K_L - K_D + k_2^{-2} (K_H - K_{MA})]$	
$J_\epsilon = -\frac{\rho d^3}{m} k_2^{-2} K_{M_\epsilon}$	
K_D	drag coefficient
K_{DA}	axial drag coefficient
K_F	Magnus force coefficient
K_H	moment coefficient due to cross angular velocity
K_L	lift force coefficient due to yaw
K_M	static moment coefficient due to yaw
K_{M_ϵ}	static moment coefficient associated with asymmetry
K_{MA}	moment coefficient due to cross acceleration
K_N	normal force coefficient due to yaw
K_{N_ϵ}	normal force coefficient associated with asymmetry
K_T	Magnus moment coefficient
$k_1 = \sqrt{\frac{A}{md^2}}$	axial radius of gyration in calibers
$k_2 = \sqrt{\frac{B}{md^2}}$	transverse radius of gyration in calibers
$M = \frac{\rho d^3}{m} k_2^{-2} K_M$	

M_2, M_3	transverse components of the aerodynamic moment
m	mass
P	ratio of local dynamic pressure to that for flight through stationary air
p	arclength along trajectory in calibers
$T =$	$\frac{\rho d^3}{m} \left[K_L - k_1^{-2} K_T \right]$
u	magnitude of velocity of missile
u_2, u_3	transverse components of velocity vector
x, y, z	distances measured in range coordinate system
$\theta =$	$\int v \, dp$, roll angle
θ_ϵ	initial orientation of force due to asymmetry
$\lambda =$	$\lambda_2 + i\lambda_3 = \frac{u_2 + iu_3}{u}$, complex yaw
λ_ϵ	magnitude of asymmetry angle
λ_{MB}	complex inclination of jet of muzzle gases
v	roll in radians per caliber of travel
$\bar{v} =$	$\frac{A}{B} v$
ρ	air density
(\cdot)	derivatives with respect to time
$(\cdot)'$	derivatives with respect to p
$(\cdot)_A$	value at the beginning of blast regime
$(\cdot)_B$	value at end of blast regime.

INTRODUCTION

For an adequately stable missile one of the most important causes of dispersion is the angle between the bore sight line and the "effective" line of departure, i.e., the jump. (The effective line of departure is defined to be the line joining the muzzle and a distant point on a gravity-free trajectory). This jump angle is determined by the impulse imparted by the gun barrel on launch (tip-off and muzzle whip), the momentum received during the blast regime (blast jump), and the influence of the aerodynamic force over the free flight trajectory (aerodynamic jump). In order to calculate blast jump and aerodynamic jump it is necessary to know the angular orientation of the missile and the rate of change of orientation at the beginning of each regime. Results are therefore given in terms of the yaw and yawing at the beginning of blast and at the beginning of free flight.

Although most of the results of this report are not original¹⁻⁴, it is felt that the derivations and the form of the results are of some interest. The introduction of aerodynamic asymmetry and yaw damping into the aerodynamic jump and spin into the blast jump may be novel features. We will not consider either tip-off or muzzle whip in this report.

AERODYNAMIC JUMP

In the introduction the aerodynamic jump was described as the third of three components of the total jump. For small angles these are additive and we will, therefore, define the aerodynamic jump to be the angle between the bore sight line and the "average" trajectory when the other contributors to jump are neglected.

In order to calculate this quantity we will make use of two coordinate systems. The first has numbered axes with the 1-axis pointing forward along the missile's axis of symmetry, the 2-axis pointing to the right in the horizontal plane, and the 3-axis pointing downward according to the right-hand rule. The second coordinate system has lettered axes with the z-axis pointing down range along the bore sight, the x-axis pointing to the left

in the horizontal plane and the y-axis pointing upward according to the right-hand rule*. For small angles the components of the aerodynamic force can be related in the following way:

$$F_z = F_1 \quad (1)$$

$$F_x + iF_y = - (F_2 + iF_3) + F_1 \left(\lambda + \frac{\dot{x} + iy}{u} \right) \quad (2)$$

where F_i are i-th components of the aerodynamic force

$\lambda = \lambda_2 + i\lambda_3$ is the complex yaw

\dot{x}, \dot{y} are x and y components of the velocity

u is the magnitude of the velocity

If only those terms of a linear force system which have a measurable effect on the motion are retained^{5,6}, we can write

$$F_1 = -\rho u^2 d^2 K_{DA} \doteq -\rho u^2 d^2 K_D \quad (3)$$

$$F_2 + iF_3 = \rho u^2 d^2 \left[(-K_N + i v K_F) \lambda + K_{N\epsilon} \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \right] \quad (4)$$

where

ρ is air density

d is diameter

v is spin in radians per caliber

$$\theta = \int_{p_B}^p v dp = v (p - p_B) \text{ for constant } v$$

θ_ϵ is initial orientation after the blast of the aerodynamic asymmetry

λ_ϵ is magnitude of the aerodynamic asymmetry.

p_B is the p coordinate at the end of the blast region and

the K_j 's are aerodynamic coefficients defined by eqs. (3-4).

* The 2 and 3 axes are the reverses of the x and y axes so that the yaw is measured from the trajectory to the missile's axis in the first system and conversely in the second.

According to Newton's equations of motion,

$$m (\dot{u}) = - \rho d^2 u^2 K_D \quad (5)$$

$$m (\ddot{x} + i\ddot{y}) = F_x + iF_y \quad (6)$$

where m is the mass

It is now convenient to change the independent variable from time to dimensionless arclength p where $\frac{dp}{dt} = \frac{u}{d}$. If primes are used to denote derivatives with respect to p ,

$$\begin{aligned} x'' + iy'' &= \left[(\dot{x} + i\dot{y}) \frac{dt}{dp} \right] \frac{dt}{dp} \\ &= \ddot{x} + i\ddot{y} \left(\frac{d}{u} \right)^2 - (\dot{x} + i\dot{y}) \left(\frac{\dot{u}d^2}{u^3} \right). \end{aligned} \quad (7)$$

From Eqs. (5) and (7) it follows that

$$m (\ddot{x} + i\ddot{y}) = \left[\frac{mu^2}{d^2} (x'' + iy'') - \rho d^2 u^2 K_D \left(\frac{\dot{x} + i\dot{y}}{u} \right) \right] \quad (8)$$

If Eqs. (2, 3, 4, 8) are substituted in Eq. (6)

$$\frac{x'' + iy''}{d} = \frac{\rho d^3}{m} \left[(K_L - i v K_F) \lambda - K_{N_e} \lambda_e e^{i(\theta + \theta_e)} \right] \quad (9)$$

where $K_L = K_N - K_D$

The aerodynamic jump can be defined analytically by the equation:

$$\text{Aerodynamic Jump} = \lim_{z \rightarrow \infty} \frac{x + iy}{z - z_B} \quad (10)$$

where z_B is z -coordinate at the end of the blast
 z_A is z -coordinate of the gun muzzle
 $z_B - z_A$ is small and
 x, y are functions of z .

By use of Eqs. (9-10) and the assumption of constant non-zero spin * and

* For symmetric missiles ($\lambda_e = 0$), Eq. (11) is valid for zero spin.

constant aerodynamic coefficients the aerodynamic jump takes on the value:

$$\begin{aligned}
 \text{Aero. Jump} &= \lim_{p \rightarrow \infty} \frac{\rho d^3}{m} \int_{p_B}^p \int_{p_B}^q \left[(K_L - i v K_F) \lambda - K_N \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \right] dr dq \\
 &= \frac{\rho d^3}{m} \left\{ \lim_{p \rightarrow \infty} \frac{1}{p - p_B} \int_{p_B}^p \int_{p_B}^q \left[(K_L - i v K_F) \right] \lambda dr dq \right\} - \left(\frac{\rho d^3}{m} \right) \left[\frac{1 K_N \lambda_\epsilon e^{i\theta_\epsilon}}{v} \right]
 \end{aligned} \quad (11)$$

The differential equation for λ can now be written with gravity neglected^{6,7,8}:

$$\lambda'' + (H - i\bar{v}) \lambda' - (M + i\bar{v}T) \lambda = J_\epsilon \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \quad (12)$$

where

$$H = \frac{\rho d^3}{m} \left[K_L - K_D + k_2^{-2} (K_H - K_{MA}) \right]$$

$$\bar{v} = \frac{A}{B} v$$

$$M = \frac{\rho d^3}{m} k_2^{-2} K_M$$

$$T = \frac{\rho d^3}{m} (K_L - k_1^{-2} K_T)$$

$$J_\epsilon = - \frac{\rho d^3}{m} k_2^{-2} K_{M_\epsilon}$$

and remaining symbols are defined in the Table of Symbols.

Eq. (12) can be solved for λ , substituted in Eq. (11), and the result integrated.

$$\begin{aligned}
 \text{Aero. Jump} &= \frac{\rho d^3}{m} \left\{ \lim_{p \rightarrow \infty} \frac{1}{p - p_B} \int_{p_B}^p \frac{(K_L - i v K_F)}{(M + i \bar{v} T)} \left[(\lambda' - \lambda'_B) + (H - i \bar{v}) (\lambda - \lambda_B) \right] dq \right\} \\
 &\quad - \frac{\rho d^3}{m} \left[\frac{(K_L - i v K_F) J_\epsilon}{M + i \bar{v} T} + K_{N_\epsilon} \right] \frac{i \lambda_\epsilon e^{i\theta_\epsilon}}{v}
 \end{aligned} \quad (13)$$

Since integrals of λ' and λ are bounded for dynamically stable missiles, Eq. (13) can be written in its final form:

$$\text{Aero. Jump} = -\frac{\rho d^3}{m} \left\{ \left[\frac{K_L - i v K_F}{M + i v T} \right] \left[\lambda_B' + (H - i v) \lambda_B \right] + \left[\frac{(K_L - i v K_F) J_\epsilon}{M + i v T} + K_{N_\epsilon} \right] \frac{i \lambda_\epsilon e^{i\theta_\epsilon}}{v} \right\} \quad (14)$$

NOTE: Although Eq. (14) is the exact relation for aerodynamic jump, it is sometimes convenient to make use of an approximation based on no aerodynamic damping and no Magnus force. ($K_F = H = T = 0$)

$$\therefore \text{Aero. Jump} = -\frac{K_L k_2^2}{K_M} (\lambda_B' - i v \lambda_B) - \frac{\rho d^3}{m} \left[K_{N_\epsilon} - \frac{K_L K_{M_\epsilon}}{K_M} \right] \frac{i \lambda_\epsilon e^{i\theta_\epsilon}}{v} \quad (15)$$

It is interesting to note that if the center of pressure of the normal force is at the same location as the center of pressure of the force due to asymmetry and $K_N = K_L$, the coefficient of λ_ϵ vanishes and the asymmetry has no effect on aerodynamic jump.

EFFECT OF BLAST ON TOTAL JUMP

The parameters which are used to describe the aerodynamic force are the diameter, the dynamic pressure,* $\frac{1}{2} \rho u^2$, the complex yaw, λ , the transverse angular velocity and the aerodynamic coefficients which are functions of Mach number. The definitions of dynamic pressure and complex yaw are in terms of flight through stationary air. Since these variables are convenient for the dynamic problem, we will want to modify the definitions so that they will apply to the blast regime when the gases surrounding the model are in motion.

In order to describe this blast regime we will make the brutal assumption that the blast can be approximated by an inclined jet of uniform flow with

* In ballistic usage the $\frac{1}{2}$ factor is omitted.

density, speed, and inclination angle varying with time. The effect of such a jet can be described by a corrected dynamic pressure and a flow inclination. Since linear aerodynamic coefficients are implicitly defined for forward flow, we will have to consider them as functions of the direction of flow as well as the Mach number of the missile with respect to the gases about it.

With this in mind we make the following definitions:

1. P is the ratio of the dynamic pressure of gases flowing past the configuration to the dynamic pressure which would be present if the configuration were traveling in still air. ($\frac{1}{2} \rho u^2 P$ is, then, the dynamic pressure of the gases flowing past the missile.)
2. $\lambda + \lambda_{MB}$ is the angle between the missile's axis and the velocity vector of the muzzle gases. (λ_{MB} is, therefore, the angle between the missile's velocity vector and the jet velocity vector.)

Actual observations of blast as well as estimates of its effect indicate that the major contribution of blast to jump is made in five to twenty calibers of travel. This means that the static moment and the moment due to asymmetry are the only moments which have an appreciable effect on the motion. Similarly the only forces which should be considered are the normal force and the force due to asymmetry.

$$\therefore F_2 + iF_3 = \rho d^2 u^2 P \left[-K_N(\lambda + \lambda_{MB}) + K_{N_e} \lambda_e e^{i(\theta + \theta_e)} \right] \quad (18)$$

$$M_2 + iM_3 = \rho d^3 u^2 P \left[-iK_M(\lambda + \lambda_{MB}) + iK_{M_e} \lambda_e e^{i(\theta + \theta_e)} \right] \quad (19)$$

Under these conditions the lift damping of the yaw can be neglected ($T = H = 0$) and the equation of yawing motion reduces to

$$\lambda'' - i\nu\lambda' - PM(\lambda + \lambda_{MB}) = PJ_e \lambda_e e^{i(\theta + \theta_e)} \quad (20)$$

The transverse deflection of trajectory is governed by the following differential equation:

$$\frac{x'' + iy''}{\bar{a}} = \frac{\rho d^3}{m} P \left[K_L (\lambda + \lambda_{MB}) - K_{N\epsilon} \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \right] \quad (21)$$

Since the blast occurs over such a small portion of the trajectory, its effect on jump is manifested in a change in the slope of the trajectory.

$$\therefore \text{Blast Jump} = \frac{(x'_B - x'_A) + i(y'_B - y'_A)}{\bar{a}} \quad (22)$$

where the A subscript denotes conditions at the beginning of blast and the B subscript denotes the conditions at the end of the blast

From Eqs. (20-22) it follows that

$$\begin{aligned} \text{Blast Jump} &= \frac{\rho d^3}{m} \int_{p_A}^{p_B} P \left[K_L (\lambda + \lambda_{MB}) - K_{N\epsilon} \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \right] dp \\ &= \int_{p_A}^{p_B} \left\{ \frac{K_L}{k_2^{-2} K_M} \left[\lambda'' - i \bar{v} \lambda' \right] + P \lambda_\epsilon e^{i(\theta + \theta_\epsilon)} \left[\frac{K_L K_{M\epsilon}}{K_M} - K_{N\epsilon} \right] \right\} dp \end{aligned} \quad (23)$$

The blast regime is characterized by the fact that most of the momentum is imparted when the projectile is in reverse flow and so we will make use of reverse flow values of the aerodynamic coefficients in Eq. (23). These coefficients will be identified by R subscripts.

Eq. (23) allows us to make two statements about the effect of aerodynamic asymmetry on blast jump:

1. For $\left[\frac{K_L}{K_M} \right]_R = \left[\frac{K_{N\epsilon}}{K_M} \right]_R$, this effect is zero if the center of pressure of the force due to asymmetry in reverse flow is at the same point as the center of pressure of normal force in reverse flow.*

$$\left[\left(\frac{K_{M\epsilon}}{K_{N\epsilon}} \right)_R \right] = \left(\frac{K_M}{K_N} \right)_R$$

* This property is quite similar to that obtained in the section on aerodynamic jump.

2. Since the blast region is so small, $\theta \approx 0$ for $p_A < p < p_B$ and the blast jump due to aerodynamic asymmetry is in the direction of the force due to asymmetry.

Turning now to the case of the symmetric missile we can make the assumption that $\left(\frac{K_L}{K_M}\right)_R$ is constant and obtain the following simple relation:

$$\text{Blast Jump} = \left(\frac{K_L}{K_M}\right)_R k_2^2 \left[(\lambda_B' - \lambda_A') - i\bar{v}(\lambda_B - \lambda_A) \right] \quad (24)$$

Adding Eq. (15) for $\lambda_e = 0$ to Eq. (24) and neglecting any transverse velocity at the gun muzzle, we can write an important and concise equation for total jump.

$$\begin{aligned} \text{Jump} = & \left[\left(\frac{K_L}{K_M}\right)_R - \frac{K_L}{K_M} \right] \left[\lambda_B' - i\bar{v}\lambda_B \right] k_2^2 \\ & - \left(\frac{K_L}{K_M}\right)_R \left[\lambda_A' - i\bar{v}\lambda_A \right] k_2^2 \end{aligned} \quad (25)$$

Eq. (25) shows that if the center of pressure in reverse flow is the same as the center of pressure in forward flow, it is not necessary to know the angular motion through the blast in order to calculate the jump.* The jump is then completely determined by conditions at launching, i.e.

$$\text{Jump} = - \left[\frac{K_L}{K_M} \right] \left[\lambda_A' - i\bar{v}\lambda_A \right] k_2^2 \quad (26)$$

Charles H. Murphy
CHARLES H. MURPHY

* This result has previously been obtained independently by both E. L. Kessler and C. L. Poor.

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